

List of Theorems

- Thm 1.A. The class of regular languages is closed under union.
- Thm 1.B. The class of regular languages is closed under concatenation.
- Thm 1.C. Every NFA has an equivalent DFA.
- Thm 1.D. The class of regular languages is closed under Kleene-star.
- Thm 1.E. (Kleene's Theorem) Language A is regular iff A has a regular expression.
- Thm 1.F. If A is finite language, then A is regular.
- Thm 1.G. The class of regular languages is closed under intersection.
- Thm 1.H. The class of regular languages is closed under complementation.
- Thm 1.I. (Pumping lemma for regular languages) If A is regular language, then \exists number p where, if $s \in A$ with $|s| \geq p$, then \exists strings x, y, z such that $s = xyz$ and (1) $xy^iz \in A$ for each $i \geq 0$, (2) $|y| > 0$, and (3) $|xy| \leq p$.
- Thm 2.A. Every CFL can be described by a CFG $G = (V, \Sigma, R, S)$ in Chomsky normal form, i.e., each rule in G has one of two forms: $A \rightarrow BC$ or $A \rightarrow x$, where $A \in V$, $B, C \in V - \{S\}$, $x \in \Sigma$, and we also allow the rule $S \rightarrow \varepsilon$.
- Thm 2.B. If A is a regular language, then A is also a CFL.
- Thm 2.C. A language is context free iff some PDA recognizes it.
- Thm 2.D. (Pumping lemma for CFLs) For every CFL L , \exists pumping length p such that \forall strings $s \in L$ with $|s| \geq p$, we can write $s = uvxyz$ with (1) $uv^ixy^iz \in L \forall i \geq 0$, (2) $|vy| \geq 1$, (3) $|vxy| \leq p$.
- Thm 2.E. The class of CFLs is closed under union.
- Thm 2.F. The class of CFLs is closed under concatenation.
- Thm 2.G. The class of CFLs is closed under Kleene-star.
- Thm 3.A. For every multi-tape TM M , there is a single-tape TM M' such that $L(M) = L(M')$.
- Thm 3.B. Every NTM has an equivalent deterministic TM.
- Cor 3.C. Language L is Turing-recognizable iff an NTM recognizes it.
- Thm 3.D. A language is enumerable iff some enumerator enumerates it.
- Church-Turing Thesis. The informal notion of algorithm is the same as Turing machine algorithm.
- Thm 4.A. $A_{\text{DFA}} = \{ \langle B, w \rangle \mid B \text{ is a DFA that accepts string } w \}$ is Turing-decidable.
- Thm 4.B. $A_{\text{NFA}} = \{ \langle B, w \rangle \mid B \text{ is an NFA that accepts string } w \}$ is Turing-decidable.
- Thm 4.C. $A_{\text{REX}} = \{ \langle R, w \rangle \mid R \text{ is a regular expression that generates string } w \}$ is Turing-decidable.
- Thm 4.D. $E_{\text{DFA}} = \{ \langle B \rangle \mid B \text{ is a DFA with } L(B) = \emptyset \}$ is Turing-decidable.
- Thm 4.E. $EQ_{\text{DFA}} = \{ \langle A, B \rangle \mid A \text{ and } B \text{ are DFAs with } L(A) = L(B) \}$ is Turing-decidable.
- Thm 4.F. $A_{\text{CFG}} = \{ \langle G, w \rangle \mid G \text{ is a CFG that generates string } w \}$ is Turing-decidable.
- Thm 4.G. $E_{\text{CFG}} = \{ \langle G \rangle \mid G \text{ is a CFG with } L(G) = \emptyset \}$ is Turing-decidable.
- Thm 4.H. Every CFL is Turing-decidable.
- Thm 4.I. $A_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a TM that accepts string } w \}$ is undecidable.
- Thm 4.J. The set \mathcal{R} of all real numbers is uncountable.

Cor 4.K. Some languages are not Turing-recognizable.

Thm 4.L. A language is decidable iff it is both Turing-recognizable and co-Turing-recognizable.

Cor 4.M. $\overline{A_{TM}}$ is not Turing-recognizable.

Thm 5.A. $HALT_{TM} = \{ \langle M, w \rangle \mid M \text{ is a TM that halts on } w \}$ is undecidable.

Thm 5.B. $E_{TM} = \{ \langle M \rangle \mid M \text{ is a TM with } L(M) = \emptyset \}$ is undecidable.

Thm 5.C. $REG_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is regular} \}$ is undecidable.

Thm 5.D. $EQ_{TM} = \{ \langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are TMs with } L(M_1) = L(M_2) \}$ is undecidable.

Thm 5.E. (Rice's Thm.) Let \mathcal{P} be any subset of the class of Turing-recognizable languages such that $\mathcal{P} \neq \emptyset$ and $\overline{\mathcal{P}} \neq \emptyset$. Then $L_{\mathcal{P}} = \{ \langle M \rangle \mid L(M) \in \mathcal{P} \}$ is undecidable.

Thm 5.F. If $A \leq_m B$ and B is Turing-decidable, then A is Turing-decidable.

Cor 5.G. If $A \leq_m B$ and A is undecidable, then B is undecidable.

Thm 5.H. If $A \leq_m B$ and B is Turing-recognizable, then A is Turing-recognizable.

Cor 5.I. If $A \leq_m B$ and A is not Turing-recognizable, then B is not Turing-recognizable.

Thm 5.J. $E_{TM} = \{ \langle M \rangle \mid M \text{ is a TM with } L(M) = \emptyset \}$ is not Turing-recognizable.

Thm 5.K. $EQ_{TM} = \{ \langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are TMs with } L(M_1) = L(M_2) \}$ is neither Turing-recognizable nor co-Turing-recognizable.

Thm 7.A. Let $t(n)$ be a function with $t(n) \geq n$. Then any $t(n)$ -time multi-tape TM has an equivalent $O(t^2(n))$ -time single-tape TM.

Thm 7.B. Let $t(n)$ be a function with $t(n) \geq n$. Then any $t(n)$ -time NTM has an equivalent $2^{O(t(n))}$ -time deterministic 1-tape TM.

Thm 7.C. $PATH \in P$.

Thm 7.D. $RELPRIME \in P$.

Thm 7.E. Every CFL is in P.

Thm 7.F. A language is in NP iff it is decided by some nondeterministic polynomial-time TM.

Cor 7.G. $NP = \bigcup_{k \geq 0} NTIME(n^k)$

Thm 7.H. $CLIQUE \in NP$.

Thm 7.I. $SUBSET-SUM \in NP$.

Thm 7.J. If $A \leq_P B$ and $B \in P$, then $A \in P$.

Thm 7.K. $3SAT$ is polynomial-time reducible to $CLIQUE$.

Thm 7.L. If there is an NP-Complete problem B and $B \in P$, then $P = NP$.

Thm 7.M. If B is NP-Complete and $B \leq_P C$ for $C \in NP$, then C is NP-Complete.

Thm 7.N. (Cook-Levin Thm.) SAT is NP-Complete.

Cor 7.O. $3SAT$ is NP-Complete.

Cor 7.P. $CLIQUE$ is NP-Complete.

Thm 7.Q. ILP is NP-Complete.